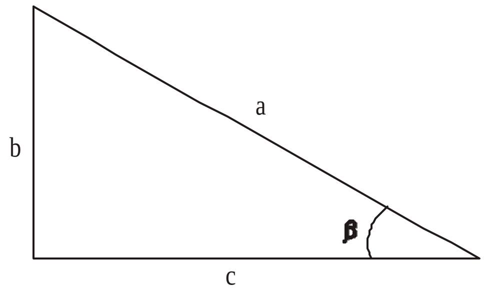
ANEXO III

RELAÇÕES TRIGONOMÉTRICAS.

Sendo dado o triângulo abaixo.



As seguintes relações são válidas:

1) $\text{sen}\,\mathbf{b} = \dfrac{b}{a}$	2) $\cos\mathbf{b} = \dfrac{c}{a}$	3) $\text{tg}\,\mathbf{b} = \dfrac{b}{c}$
4) $\text{cosec}\,\mathbf{b} = \dfrac{a}{b}$	5) $\sec\mathbf{b} = \dfrac{a}{c}$	6) $\text{cotg}\,\mathbf{b} = \dfrac{c}{b}$
7) $\text{tg}\,\mathbf{b} = \dfrac{\text{sen}\,\mathbf{b}}{\cos\mathbf{b}}$	8) $\text{sen}^2\,\mathbf{b} + \cos^2\,\mathbf{b} = 1$	9) $\sec^2\,\mathbf{b} - 1 = \text{tg}^2\,\mathbf{b}$
10) $\text{sen}(\mathbf{a} \pm \mathbf{b}) = \text{sen}\,\mathbf{a}\,\cos\mathbf{b} \pm \cos\mathbf{a}\,\text{sen}\,\mathbf{b}$	11) $\cos(\mathbf{a} \pm \mathbf{b}) = \cos\mathbf{a}\,\cos\mathbf{b} \mp \text{sen}\,\mathbf{a}\,\text{sen}\,\mathbf{b}$	
12) $\text{sen}\,\mathbf{a} \pm \text{sen}\,\mathbf{b} = 2\cdot\text{sen}\dfrac{1}{2}(\mathbf{a} \pm \mathbf{b})\,\cos\dfrac{1}{2}(\mathbf{a} \mp \mathbf{b})$		
13) $\cos\mathbf{a} + \cos\mathbf{b} = 2\cdot\cos\dfrac{1}{2}(\mathbf{a} + \mathbf{b})\,\cos\dfrac{1}{2}(\mathbf{a} - \mathbf{b})$		
14) $\cos\mathbf{a} - \cos\mathbf{b} = -2\cdot\text{sen}\dfrac{1}{2}(\mathbf{a} + \mathbf{b})\,\text{sen}\dfrac{1}{2}(\mathbf{a} - \mathbf{b})$		
15) $\text{sen}\,\mathbf{a}\,\text{sen}\,\mathbf{b} = \dfrac{1}{2}\cdot\left[\cos(\mathbf{a} - \mathbf{b}) - \cos(\mathbf{a} + \mathbf{b})\right]$		
16) $\cos\mathbf{a}\,\cos\mathbf{b} = \dfrac{1}{2}\cdot\left[\cos(\mathbf{a} - \mathbf{b}) + \cos(\mathbf{a} + \mathbf{b})\right]$		
17) $\text{sen}\,\mathbf{a}\,\cos\mathbf{b} = \dfrac{1}{2}\cdot\left[\text{sen}(\mathbf{a} - \mathbf{b}) + \text{sen}(\mathbf{a} + \mathbf{b})\right]$		
18) $\text{sen}\,2\,\mathbf{a} = 2\,\text{sen}\,\mathbf{a}\,\cos\mathbf{a}$		
19) $\cos 2\alpha = \cos^2\alpha - \text{sen}^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\text{sen}^2\alpha$		
20) $\text{sen}^2\,\dfrac{1}{2}\mathbf{a} = \dfrac{1}{2}(1 - \cos\mathbf{a})$	21) $\cos^2\,\dfrac{1}{2}\mathbf{a} = \dfrac{1}{2}(1 + \cos\mathbf{a})$	

RELAÇÕES LOGARÍTIMICAS E EXPONENCIAIS.

1) $e^x \cdot e^y = e^{(x+y)}$	2) $(e^x)^y = e^{x \cdot y} = (e^y)^x$
3) $\log x \cdot y = \log x + \log y$	4) $\log \frac{x}{y} = \log x - \log y$
5) $\log x^a = a \cdot \log x$	6) $\ln x = (\ln 10) \cdot \log x = 2{,}3026 \cdot \log x$
7) $\log x = \log e \ \ln x = 0{,}43429 \cdot \ln x$	

NÚMEROS COMPLEXOS.

Definimos $j^2 = -1$ ou $j = \sqrt{-1}$

$$e^{\pm j\theta} = \cos \theta \pm j \operatorname{sen} \theta$$

$$\cos \theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}), \qquad \operatorname{sen} \theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

SÉRIE DE TAYLOR.

$$f(x) = f(x_0) + \frac{(x-x_0)}{1!}\left(\frac{\partial f}{\partial x}\right)\bigg|_{x=x_0} + \frac{(x-x_0)^2}{2!}\left(\frac{\partial^2 f}{\partial x^2}\right)\bigg|_{x=x_0} + \ldots \qquad \ldots. + \frac{(x-x_0)^n}{n!}\left(\frac{\partial^n f}{\partial x^n}\right)\bigg|_{x=x_0}$$

SÉRIES COMUMENTE USADAS.

1) $\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$
2) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$
3) $\operatorname{tg} x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \ldots$ 4) $e^{ax} = 1 + ax + \frac{a^2x^2}{2!} + \frac{a^3x^3}{3!} + \ldots$
5) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$
6) $(1+x)^n = 1 + nx + \frac{n(n-1)\,x^2}{2!} + \frac{n(n-1)\cdot(n-2)\,x^3}{3!} + \ldots$

A expressão 6 é verificada para qualquer n e x sendo satisfeita a relação seguinte $x^2 < 1$

DERIVADAS E INTEGRAIS.

Propriedades.

Linearidade.

1) $\frac{d}{dt}\left[A \cdot f(t)\right] = A\frac{df(t)}{dt}$ 2) $\frac{d}{dt}\left[f(t) + p(t)\right] = \frac{df(t)}{dt} + \frac{dp(t)}{dt}$

Derivada de um produto.

Sendo, f(t) e g(t) duas funções temos que:

$$\frac{d}{dt}\left[f(t) \cdot g(t)\right] = f(t) \cdot \frac{dg(t)}{dt} + \frac{df(t)}{dt} \cdot g(t)$$

Derivada de um quociente.

Sendo f(t) e g(t) duas funções, temos que:

$$\frac{d}{dt}\left[\frac{f(t)}{g(t)}\right] = \frac{\frac{df(t)}{dt} \cdot g(t) - \frac{dg(t)}{dt} \cdot f(t)}{\left[g(t)\right]^2}$$

Tabela de Derivadas e Integrais

$f(u)$	$df(u)/du$	$\int f(u)du$
u^n	$nu^{n-1}u'$	$u^{n+1}/n+1 + C \qquad (n \neq -1)$
u^{-1}	$-(1/u^2)u'$	$\ln u + C$
$\ln u$	$(1/u)\ u'$	$u \ln u - u + C$
e^u	$e^u u'$	$e^u + C$
$\text{sen } u$	$\cos u\ u'$	$-\cos u + C$
$\cos u$	$-\text{sen}^2\ u\ u'$	$\text{sen } u + C$
$\text{tg } u$	$\sec^2 u\ u'$	$-\ln \cos u + C$
$\text{cotg } u$	$-\text{cosc } u\ u'$	$\ln \text{sen } u + C$
$\text{senh } u$	$\cosh u\ u'$	$\text{senh } u + C$
$\cosh u$	$\text{senh } u\ u'$	$\cosh u + C$